

Work in Progress:

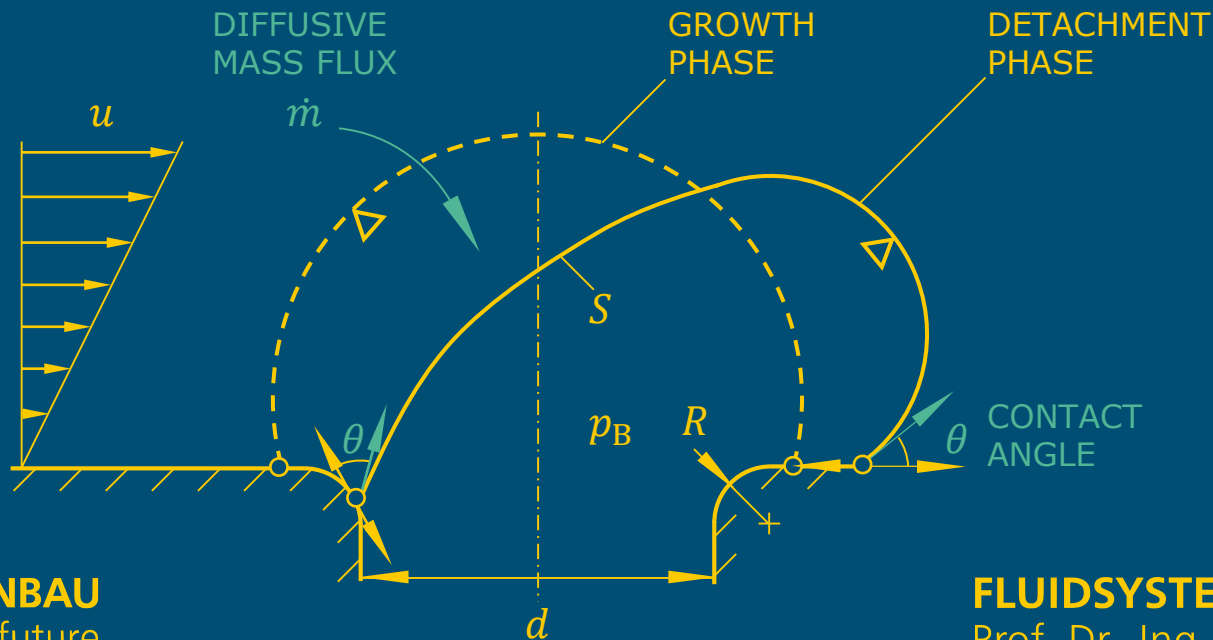
Modellierung der Ablösung von Porenkeimen und des Diffusionsprozesses bei Spitzenwirbelkavitation



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Workshop Kavitation, Drübeck, 30. November 2021

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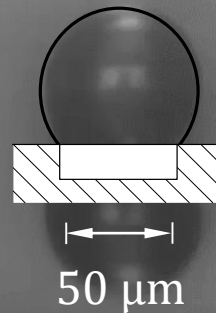


Diffusionsgetriebene Keimbildung

Aufnahme: 3000 fps
Video: 30 fps

$$\mu = 1 \text{ mPa s}$$

$$\bar{U} = 0.15 \text{ m/s}$$

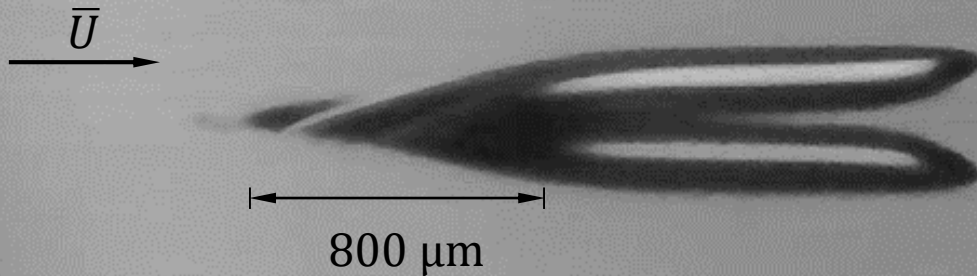
Ablösung infolge Plateau-Rayleigh-Instabilität

Aufnahme 1000 fps
Video 15 fps

$$\mu = 18.9 \text{ mPa s}$$

$$We = 213$$

$$f = 2.5 \text{ Hz}$$



Randelementmethode

(i) *Laplacesche
Gleichung*

$$\Delta\phi = 0$$

(ii) *Integral-
gleichung*

$$\phi(\vec{x}) = \underbrace{\frac{1}{2\pi} \int_S e(\vec{x}') \ln r \, dS}_{\text{QUELLE}} - \underbrace{\frac{1}{2\pi} \int_S m(\vec{x}') \frac{\vec{n} \cdot (\vec{x} - \vec{x}')}{r^2} \, dS}_{\text{DIPOL}}$$

(iii) *Diskretisierung
des Randes*

$$\int_S (\dots) \, dS = \sum_i \underbrace{\int_{S_i} (\dots) \, dS}_{\text{PANEL } i}$$

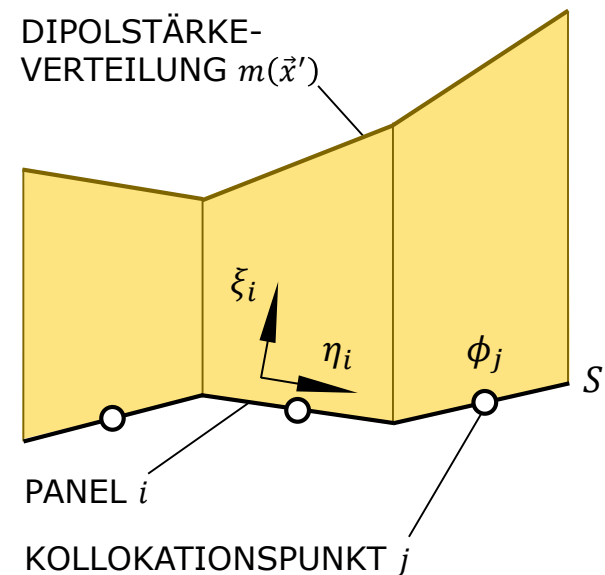
Randelementmethode

(iv) *Ansatzfunktion*

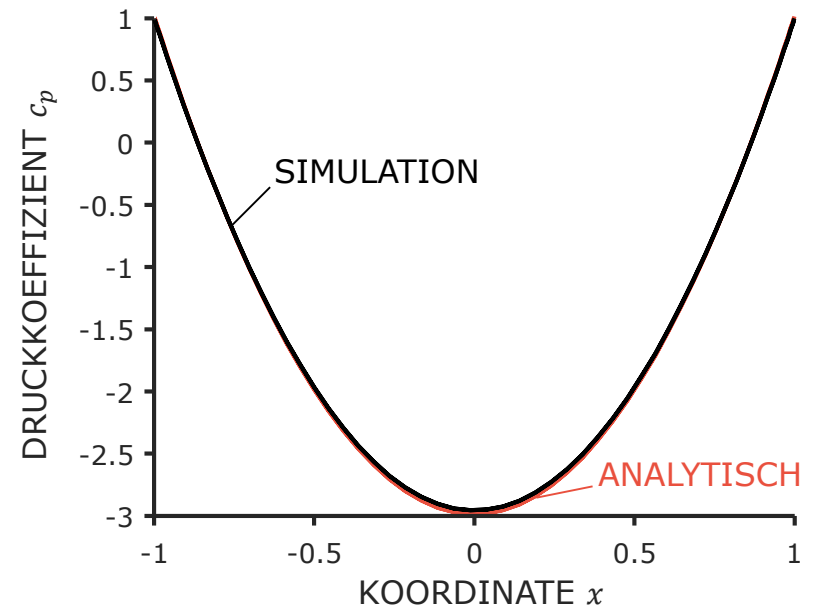
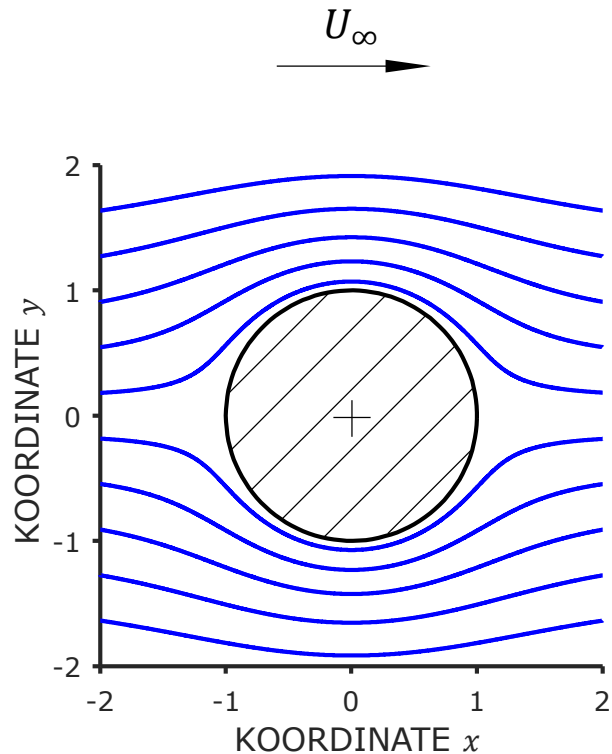
$$m_i \propto \eta_i$$

(v) *lineares
Gleichungssystem*

$$B_{ji} m_i = \phi_j$$

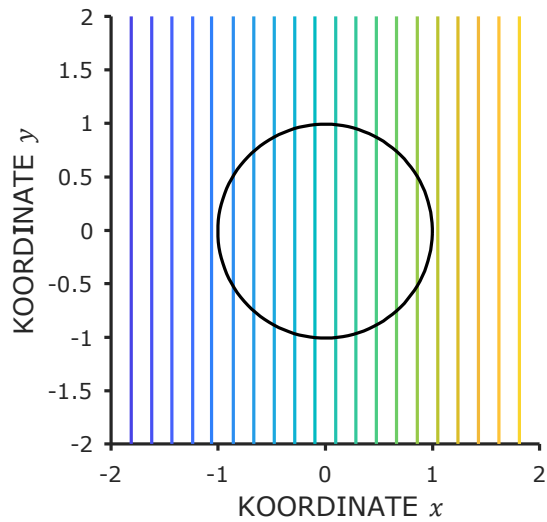


Umströmter Zylinder

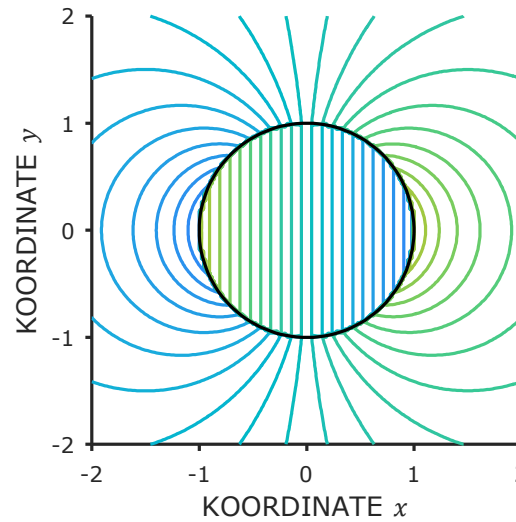


Potentialfeld

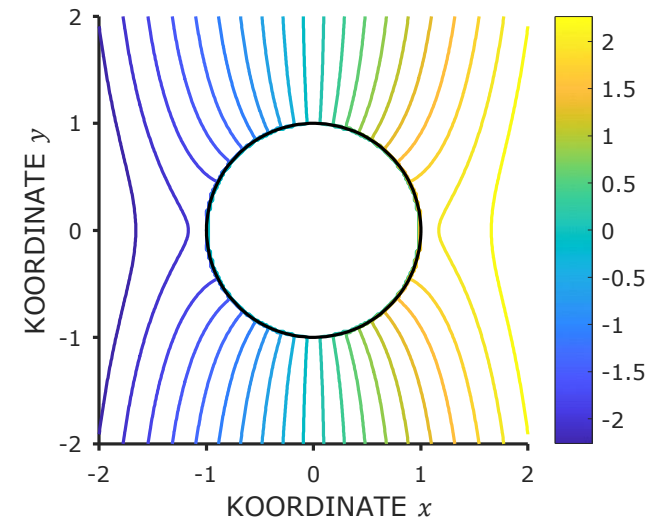
PARALLELSTRÖMUNG



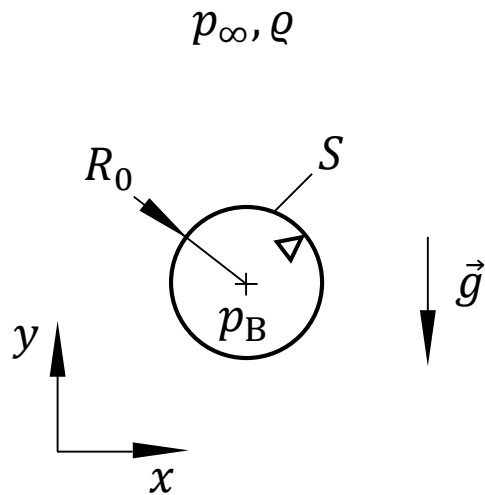
+ ZYLINDER



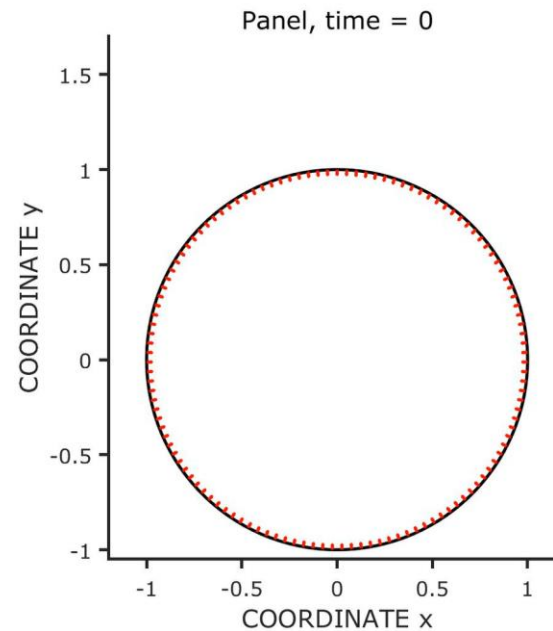
= GESAMT



Verifizierung: Instationär aufsteigende Blase



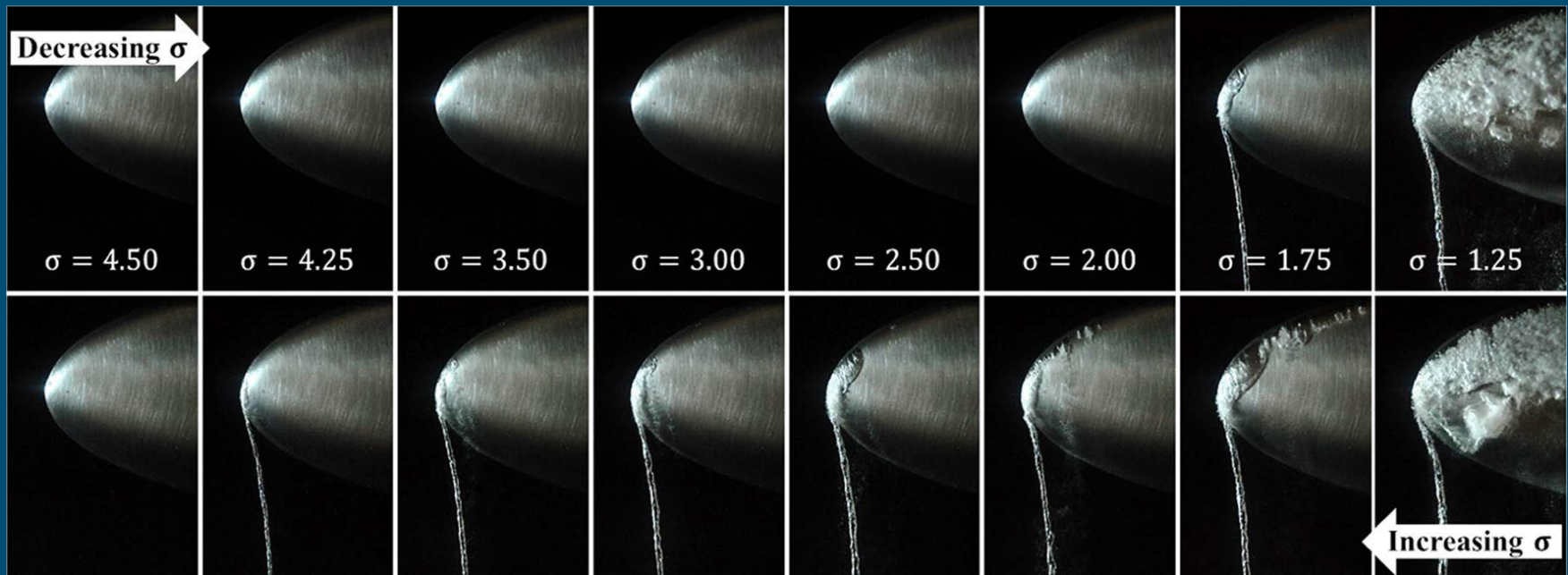
$$Bo := \frac{\rho g R_0^2}{S} = 13.25$$



Diffusionsgetriebene Hysterese bei Spitzenwirbelkavitation

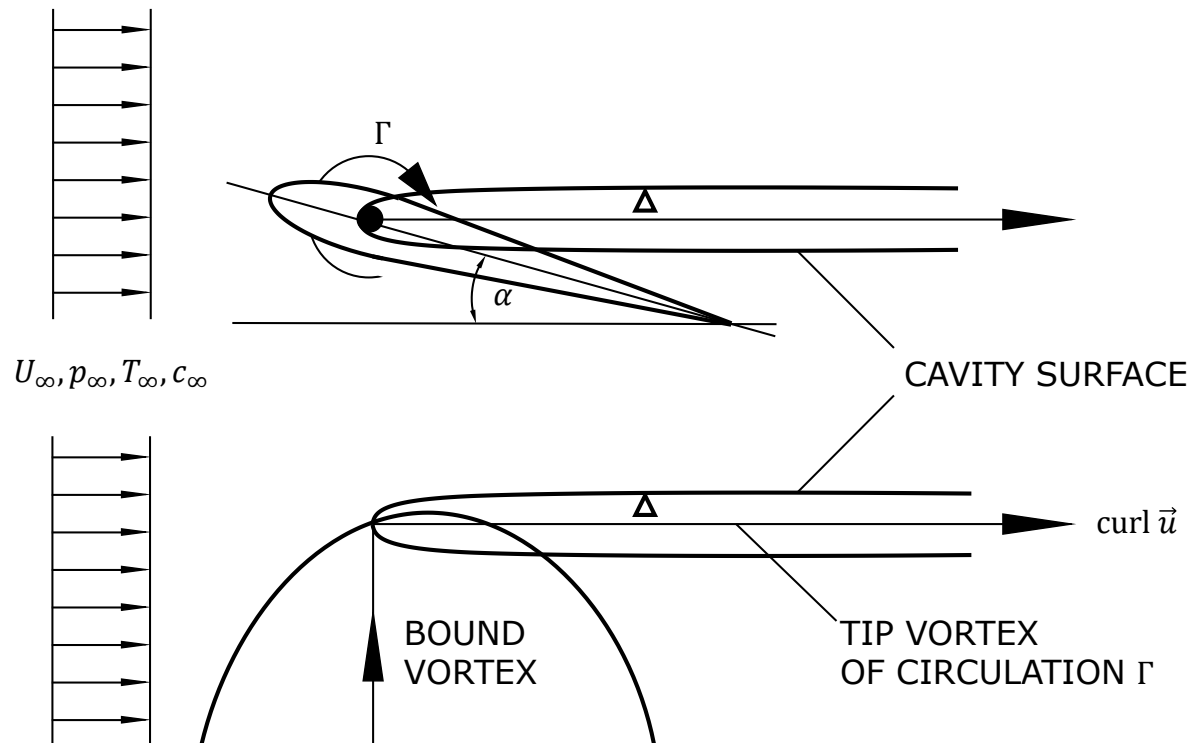


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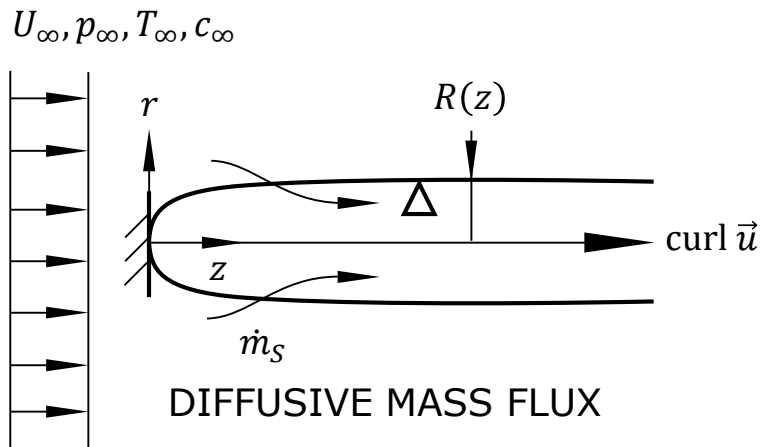
Amini et al.: *On the physical mechanism of tip vortex cavitation hysteresis*. Experiments in Fluids, 2019.

Flüssigkeitsströmung



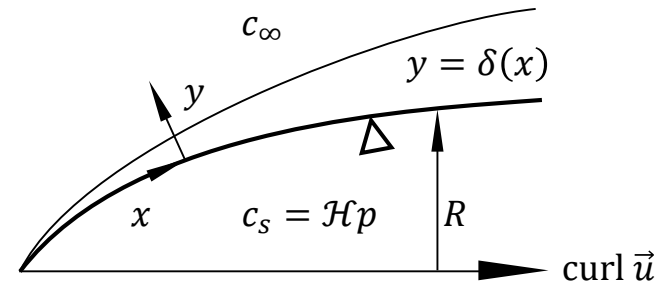
$$\vec{e}_z = \frac{\text{curl } \vec{u}}{|\text{curl } \vec{u}|}$$

Diffusiver Massentransport



$$\frac{d\dot{m}_S}{dx} = 2\pi R M_g \mathcal{D} \left. \frac{\partial c}{\partial y} \right|_{y=0}$$

$$\left. \frac{\partial c}{\partial y} \right|_{y=0} = \frac{1}{\sqrt{\pi}} (c_\infty - c_S) \sqrt{\frac{U_\infty}{x\mathcal{D}}}$$



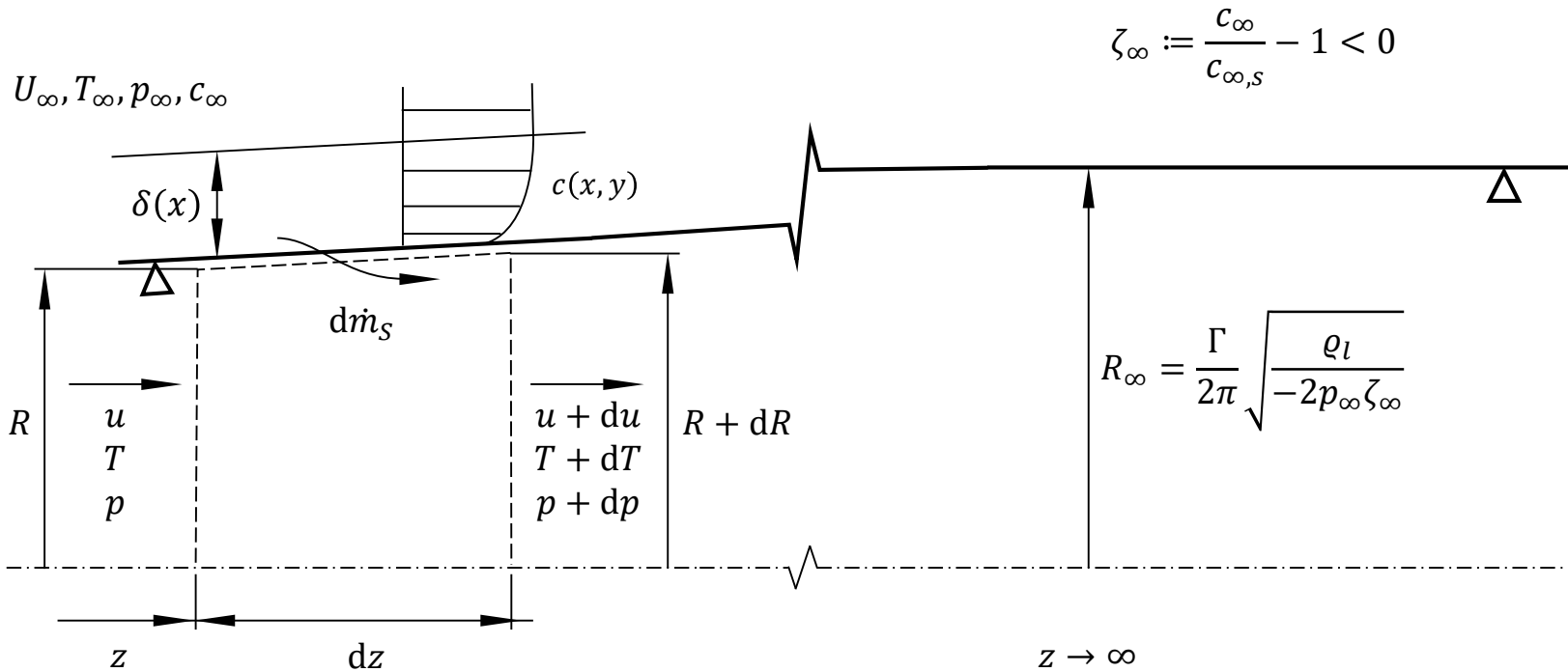
CONCENTRATION
BOUNDARY LAYER

$$f := \frac{c - c_S}{c_\infty - c_S}$$

$$U_\infty \frac{\partial f}{\partial x} = \mathcal{D} \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial \eta^2} + \frac{\eta}{2} \frac{\partial f}{\partial \eta} = 0$$

Gasströmung



Gekoppeltes System nichtlinearer gewöhnlicher Differentialgleichungen

$$\frac{du}{dz} = \frac{1}{\pi R^2 \varrho} \frac{d\dot{m}_S}{dz} - 2u \frac{1}{R} \frac{dR}{dz} - u \frac{1}{\varrho} \frac{d\varrho}{dz}$$

$$\frac{d\varrho}{dz} = \frac{1}{\pi R^2 u} \left(2 - \frac{U_\infty}{u} \right) \frac{d\dot{m}_S}{dz} + \frac{1}{u^2} \frac{dp}{dz} - \frac{2}{R} \left(\varrho - \frac{p}{u^2} \right) \frac{dR}{dz}$$

$$\frac{dp}{dz} = \varrho_l \left(\frac{\Gamma}{2\pi} \right)^2 \frac{1}{R^3} \frac{dR}{dz}$$

$$\frac{dR}{dz} = \left[1 + \frac{\varrho_l}{2} \left(\frac{\Gamma}{2\pi R} \right)^2 \frac{1}{p} \right]^{-1} \left[\frac{c_p}{\mathcal{R}} \left(1 - \frac{p}{\varrho u^2} \right) - 1 \right]^{-1} \left[\frac{1}{2} + \frac{c_p T_\infty}{u^2} + \frac{c_p}{\mathcal{R}} \left(\frac{p}{\varrho u^2} - \frac{p U_\infty}{\varrho u^3} \right) - \frac{U_\infty}{u} \right] \frac{u}{2\pi R p} \frac{d\dot{m}_S}{dz}$$

$$\frac{d\dot{m}_S}{dz} = 2RM_g (c_\infty - \mathcal{H}p) \sqrt{\frac{\pi \mathcal{D} U_\infty}{z}}$$

Zusammenfassung



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SPITZENWIRBELKAVITATION

- Erweiterung des physikalischen Modells (Wirbelströmung, Schubspannung)
- eigenes Experiment

ABLÖSUNG VON PORENKEIMEN

- Überarbeitung der BEM-Implementierung hinsichtlich Numerik
- Anwendung auf Porenkeim(e)